MONASH University

MONASH BUSINESS SCHOOL

ETF3231/5231 Business forecasting

Week 11: Dynamic Regression https://bf.numbat.space/

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Outline

Outline

* start from us-gasoline example from week10.R

Regression with ARIMA errors

Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t,$$

- y_t modeled as function of k explanatory variables $x_{1,t}, \dots, x_{k,t}$.
- In regression, we assume that ε_t is WN.
- Now we want to allow ε_t to be autocorrelated.

Regression with ARIMA errors

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- In regression, we assume that ε_t is WN.
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Example: ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

where ε_t is white noise.

Residuals and errors

Example: η_t = ARIMA(1,1,1)

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t, \qquad \text{regression}$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t, \qquad \text{innovation}$$

Residuals and errors

Example: η_t = ARIMA(1,1,1)

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$

regression

$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

innovation

- Be careful in distinguishing η_t from ε_t.
- Only the errors ε_t are assumed to be white noise.
- In ordinary regression, η_t is assumed to be white noise and so $\boxed{\eta_t = \varepsilon_t}$.

If we had no dynamics

Estimation

If we minimize $\sum \eta_t^2$ (by using ordinary regression):

1. Estimated coefficients $\hat{\beta}_0, \ldots, \hat{\beta}_k$ are no longer optimal as some information ignored;
   Consistent but not efficient
2. Statistical tests associated with the model (e.g., t-tests on the coefficients) are incorrect.
3. p-values for coefficients usually too small ("spurious regression'').
   Beware of inference
4. AIC of fitted models misleading.

Estimation

If we minimize $\sum \eta_t^2$ (by using ordinary regression):

1. Estimated coefficients $\hat{\beta}_0, \ldots, \hat{\beta}_k$ are no longer optimal as some information ignored;
2. Statistical tests associated with the model (e.g., t-tests on the coefficients) are incorrect.
3. *p*-values for coefficients usually too small ("spurious regression'').
4. AIC of fitted models misleading.

- Minimizing $\sum \varepsilon_t^2$ avoids these problems.
- Maximizing likelihood similar to minimizing $\sum \varepsilon_t^2$.

this is what we do

Stationarity

start with the simplest case

Regression with ARMA errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$

where η_t is an ARMA process.

- All variables in the model must be stationary.
- If we estimate the model while any of these are non-stationary, the estimated coefficients can be incorrect.

  unless they are cointegrated

  ETF3200, ETF5200, ETC3450
  ETF5320 ETC5345
- Difference variables until all stationary.
- If necessary, apply same differencing to all variables.

(see example that in Views)

Stationarity

Model with ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$ non-stationary

$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

Stationarity

Model with ARIMA(1,1,1) errors

model in levels

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$

non-stationary

$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

Equivalent to model with ARIMA(1,0,1) errors

model in differences

$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t,$$

$$(1 - \phi_1 B)\eta'_t = (1 + \theta_1 B)\varepsilon_t,$$

note coefficients stay the same.

differenced error

where $y'_t = y_t - y_{t-1}$, $x'_{t,i} = x_{t,i} - x_{t-1,i}$ and $\eta'_t = \eta_t - \eta_{t-1}$.

R will take care of this

Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

generalising

Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

Original data

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t$$

$$\text{where} \quad \phi(B)(1 - B)^d \eta_t = \theta(B)\varepsilon_t$$

Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

Original data

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t$$

$$\text{where} \quad \phi(B)(1-B)^d \eta_t = \theta(B)\varepsilon_t$$

After differencing all variables

We estimate these and map back

$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t.$$

d- lots of differencing

where $\phi(B)\eta'_t = \theta(B)\varepsilon_t$,

$y'_t = (1-B)^d y_t, \quad x'_{i,t} = (1-B)^d x_{i,t}, \quad$ and $\eta'_t = (1-B)^d \eta_t$

Regression with ARIMA errors

- In R, we can specify an ARIMA(p, d, q) for the errors, and d levels of differencing will be applied to all variables ($y, x_{1,t}, \dots, x_{k,t}$) during estimation. ARIMA(y ~ $x_1 + x_2$ + ~~pdq~~( )) automated
- Check that ε_t series looks like white noise.
- AICc can be calculated for final model. — still cannot compare for models with different d
- Repeat procedure for all subsets of predictors to be considered, and select model with lowest AICc value. — we write model in levels but it is estimated in differences (if required)

Forecasting

- To forecast a regression model with ARIMA errors, we need to forecast the regression part of the model and the ARIMA part of the model and combine the results.
- Some predictors are known into the future (e.g., time, dummies).
- Separate forecasting models may be needed for other predictors.
- Forecast intervals ignore the uncertainty in forecasting the predictors.

Outline

Dealing with , trend , seasonality & dynamics . In tutes :

- fourier + piecewise
- piecewise + ARIMA
- fourier + piecewise + ARIMA

Dynamic harmonic regression

* used a lot for consulting

Combine Fourier terms with ARIMA errors

Advantages

- it allows any length seasonality; * ED presentations for Peninsula Health (weekly)
- for data with more than one seasonal period, you can include Fourier terms of different frequencies; (daily, weekly, annual)
- the seasonal pattern is smooth for small values of K (but more wiggly seasonality can be handled by increasing K);
- the short-term dynamics are easily handled with a simple ARMA error.

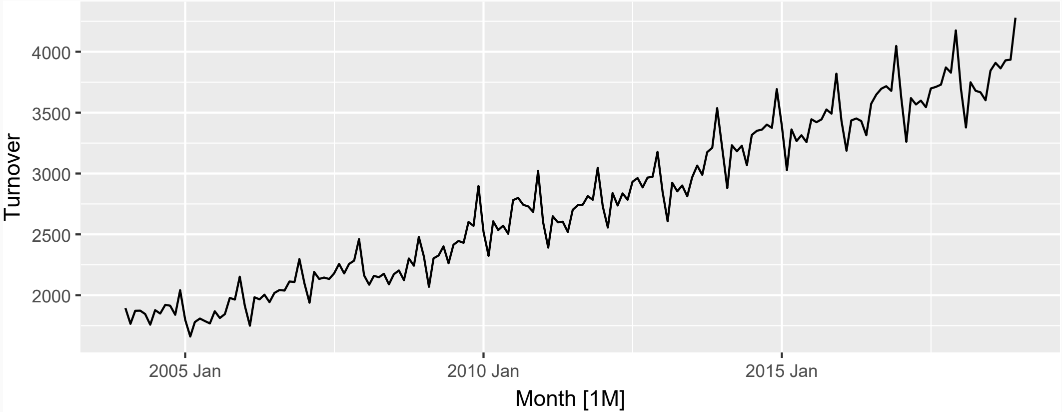
* mix & match Fourier for longer, ARIMA for shorter

Disadvantages

- seasonality is assumed to be fixed

Eating-out expenditure

```r
aus_cafe <- aus_retail |> filter(
  Industry == "Cafes, restaurants and takeaway food services",
  year(Month) %in% 2004:2018
  ) |> summarise(Turnover = sum(Turnover))
aus_cafe |> autoplot(Turnover)
```



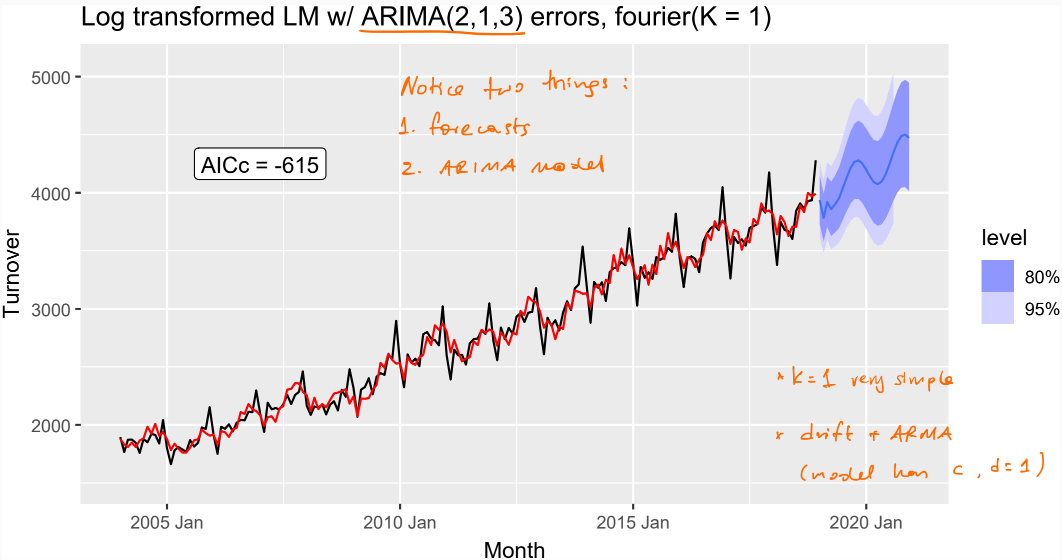
Eating-out expenditure

```r
fit <- aus_cafe |> model(
    `K = 1` = ARIMA(log(Turnover) ~ fourier(K = 1) + PDQ(0,0,0)),
    `K = 2` = ARIMA(log(Turnover) ~ fourier(K = 2) + PDQ(0,0,0)),
    `K = 3` = ARIMA(log(Turnover) ~ fourier(K = 3) + PDQ(0,0,0)),
    `K = 4` = ARIMA(log(Turnover) ~ fourier(K = 4) + PDQ(0,0,0)),
    `K = 5` = ARIMA(log(Turnover) ~ fourier(K = 5) + PDQ(0,0,0)),
    `K = 6` = ARIMA(log(Turnover) ~ fourier(K = 6) + PDQ(0,0,0))
)

glance(fit) |> select(.model, sigma2, log_lik, AIC, AICc, BIC)
```

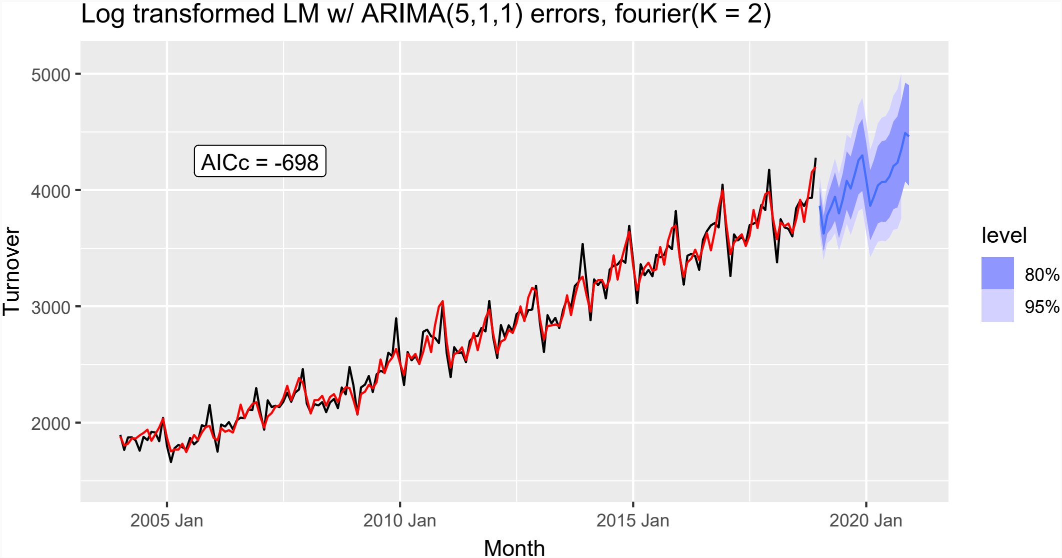
.model	sigma2	log_lik	AIC	AICc	BIC
K = 1	0.002	317	-616	-615	-588
K = 2	0.001	362	-700	-698	-661
K = 3	0.001	394	-763	-761	-725
K = 4	0.001	427	-822	-818	-771
K = 5	0.000	474	-919	-917	-875
K = 6	0.000	474	-920	$\boxed{-918}$	-875

these are very close

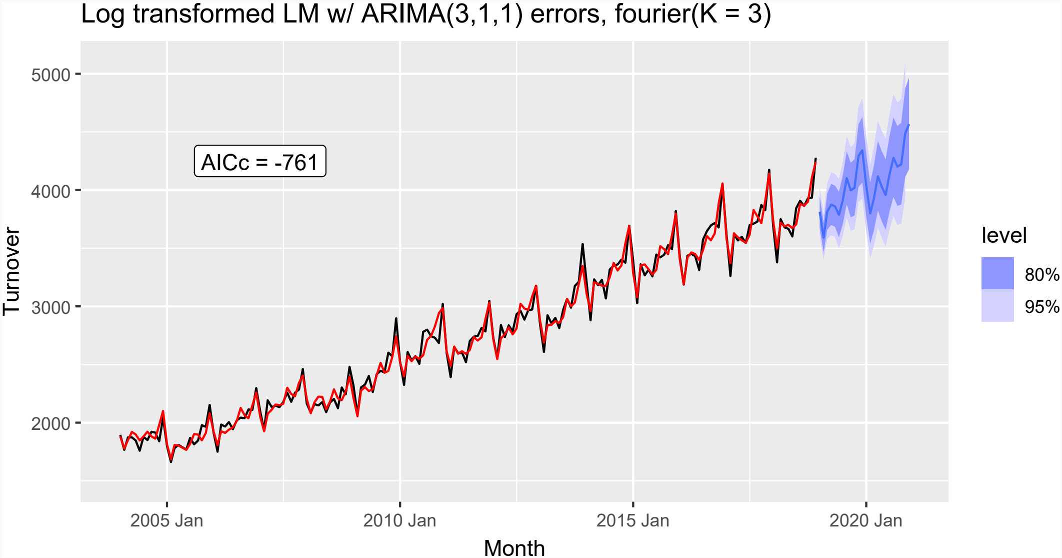
Eating-out expenditure



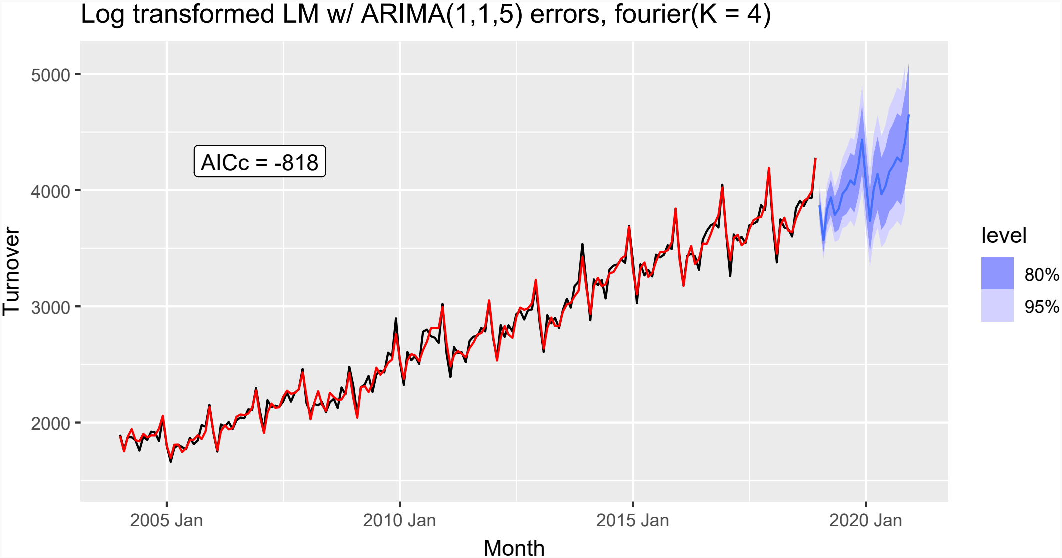
Eating-out expenditure



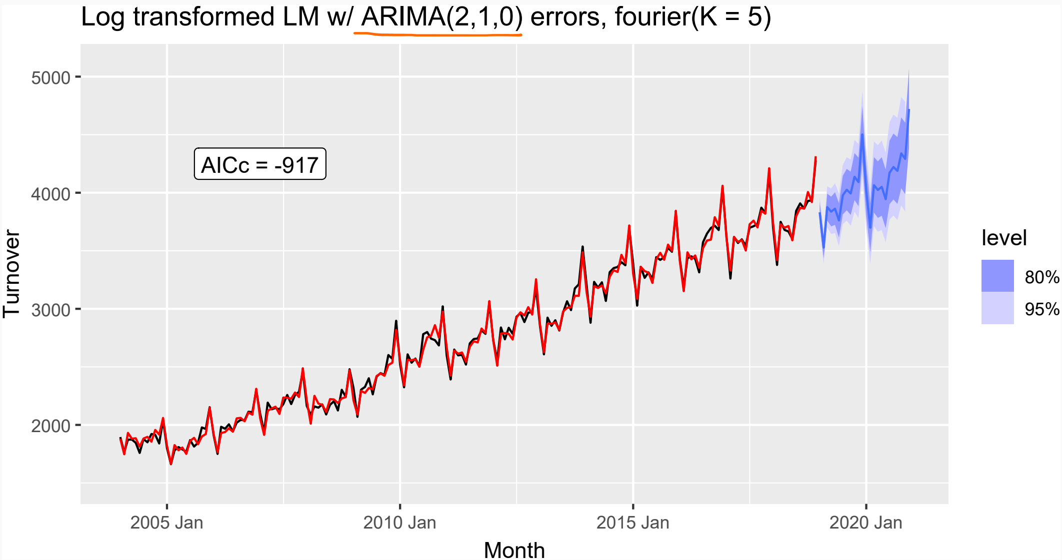
Eating-out expenditure



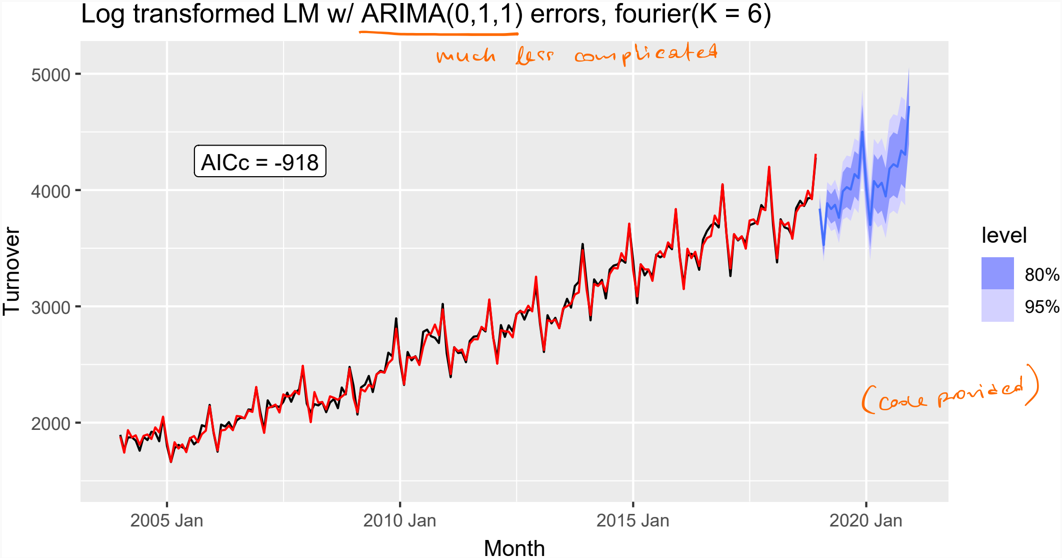
Eating-out expenditure



Eating-out expenditure



Eating-out expenditure



Outline

1. Regression with ARIMA errors
2. Dynamic harmonic regression
3. Stochastic and deterministic trends

(recall rw random)

Two ways of modelling trend that give you different results

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process. Hence stationary

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

$\eta_t \sim ARIMA(p, 0, q)$

where η_t is ARMA process.

Stochastic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

$\eta_t \sim ARIMA(p, 1, q)$

hence non-stationary

where η_t is ARIMA process with $d = 1$.

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process.

Stochastic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARIMA process with $d = 1$.
Difference both sides until η_t is stationary:

ASIDE:

$y_{t-1} = \beta_0 + \beta_1(t-1) + \eta_{t-1}$

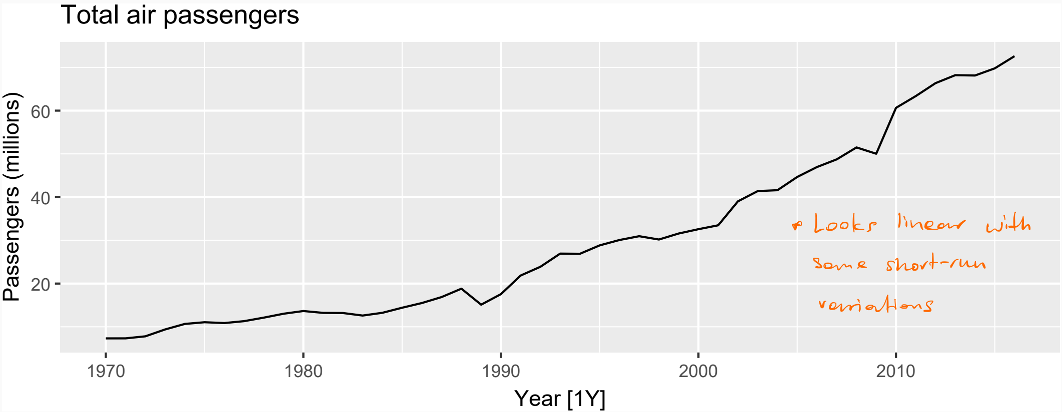
$$y'_t = \beta_1 + \eta'_t \implies \boxed{y_t = \beta_1 + y_{t-1} + \eta'_t}$$

where η'_t is ARMA process.

$\eta'_t \sim ARIMA(p, 0, q)$

Air transport passengers Australia

```
aus_airpassengers |>
  autoplot(Passengers) +
  labs(y = "Passengers (millions)",
       title = "Total air passengers")
```



Air transport passengers Australia

Deterministic trend

```
fit_deterministic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + trend() + pdq(d = 0)))
report(fit_deterministic)
```

stationary ARMA(p,q)

```
Series: Passengers
Model: LM w/ ARIMA(1,0,0) errors

Coefficients:
         ar1  trend()  intercept
      0.9564    1.415      0.901
s.e.  0.0362    0.197      7.075

sigma^2 estimated as 4.343:  log likelihood=-101
AIC=210   AICc=211   BIC=217
```

Air transport passengers Australia

Deterministic trend

```
fit_deterministic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + trend() + pdq(d = 0)))
report(fit_deterministic)
```

```
Series: Passengers
Model: LM w/ ARIMA(1,0,0) errors

Coefficients:
         ar1  trend()  intercept
      0.9564    1.415      0.901
s.e.  0.0362    0.197      7.075

sigma^2 estimated as 4.343:  log likelihood=-101
AIC=210   AICc=211   BIC=217
```

$$y_t = 0.901 + 1.415t + \eta_t$$
$$\eta_t = 0.956\eta_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim \text{NID}(0, 4.343).$$

Air transport passengers Australia

Stochastic trend

```
fit_stochastic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + pdq(d = 1)))
report(fit_stochastic)
```

non-stationary ARIMA(p,1,q)

```
Series: Passengers
Model: ARIMA(0,1,0) w/ drift

Coefficients:
      constant
         1.419
s.e.     0.301

sigma^2 estimated as 4.271:  log likelihood=-98.2
AIC=200   AICc=201   BIC=204
```

Air transport passengers Australia

Stochastic trend

```
fit_stochastic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + pdq(d = 1)))
report(fit_stochastic)
```

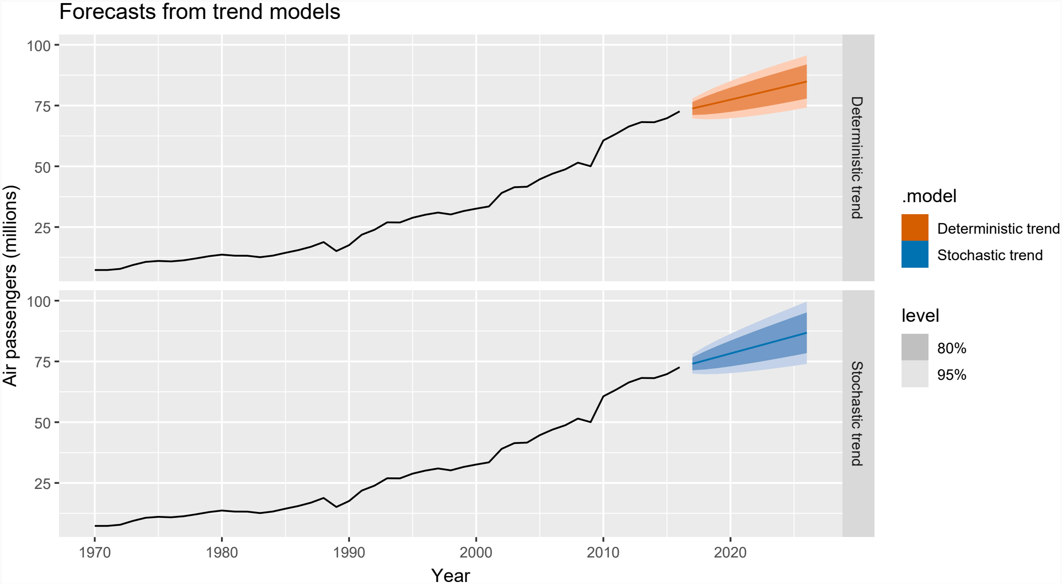
```
Series: Passengers
Model: ARIMA(0,1,0) w/ drift

Coefficients:
      constant
         1.419
s.e.     0.301

sigma^2 estimated as 4.271:  log likelihood=-98.2
AIC=200   AICc=201   BIC=204
```

similar slop coefficients

$$
\begin{aligned}
y_t - y_{t-1} &= 1.419 + \varepsilon_t, \\
y_t &= y_0 + \boxed{1.419t} + \eta_t \\
\eta_t &= \eta_{t-1} + \varepsilon_t \\
\varepsilon_t &\sim \mathrm{NID}(0, 4.271).
\end{aligned}
$$

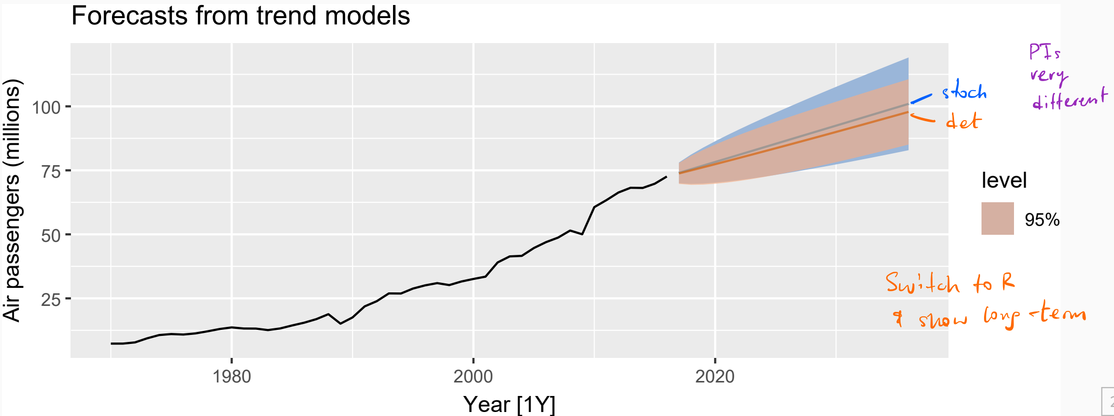
equivalent

Air transport passengers Australia



Air transport passengers Australia

```
aus_airpassengers |> autoplot(Passengers) +
  autolayer(fit_stochastic |> forecast(h = 20), colour = "#0072B2", level = 95) +
  autolayer(fit_deterministic |> forecast(h = 20), colour = "#D55E00", level = 95,
            alpha = 0.65) +
  labs(y = "Air passengers (millions)", title = "Forecasts from trend models")
```



Switch to R
& show long-term

Forecasting with trend

- Point forecasts are almost identical, but prediction intervals differ.
- Stochastic trends have much wider prediction intervals because the errors are non-stationary.
- Be careful of forecasting with deterministic trends too far ahead.

(stochastic probably the safer option)

* IA1 *