MONASH University

MONASH BUSINESS SCHOOL

# ETF3231/5231 Business forecasting

## Week 7: ARIMA models

https://bf.numbat.space/

- ETS v ARIMA
- VETS · VARIMA
- philosophy (comp v autocorv) — resids WN
- general v interpretable

AACSB ACCREDITED EQUIS ACCREDITED ASSOCIATION OF MBAS AMBA ACCREDITED

# Outline


1 Stationarity and differencing

2 Backshift notation

# ARIMA models

**AR**: autoregressive (lagged observations as inputs)

**I**: integrated (differencing to make series stationary)

**MA**: moving average (lagged errors as inputs) → not smoothing

# ARIMA models

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# ARIMA models

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**I:** integrated (differencing to make series stationary)

**MA:** moving average (lagged errors as inputs)

An ARIMA model is rarely interpretable in terms of visible data structures like trend and seasonality. But it can capture a huge range of time series patterns.

Make data stationary (variance & mean), fit model, reverse, forecast.

# Outline

ARIMA(p, d, q)(P, D, Q)

1 Stationarity and differencing I(d)(D)

2 Backshift notation

# Stationarity

**Definition**

If $\{y_t\}$ is a stationary time series, then for all $s$, the distribution of $(y_t, \dots, y_{t+s})$ does not depend on $t$.

A stationary series is:

* Think about distributions, mean variance.

- roughly horizontal
- constant variance
- no patterns predictable in the long-term

# Stationarity

## Definition

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A stationary series is:

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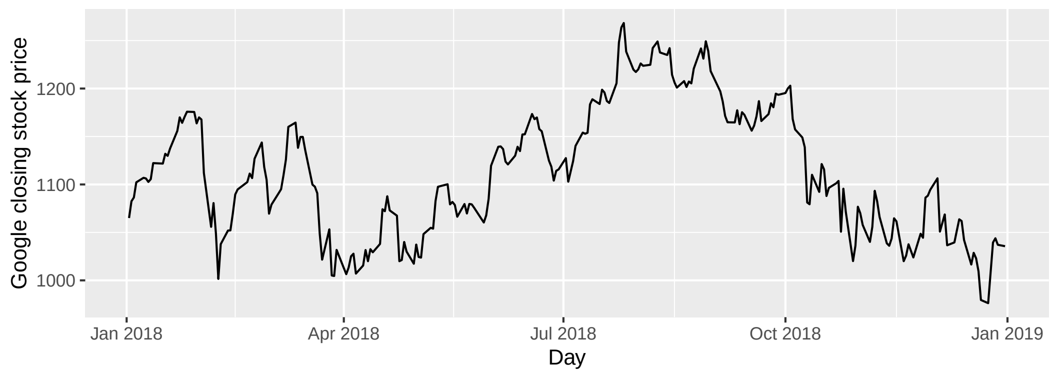
- constant variance
- no patterns predictable in the long-term

AIM: - make data stationary
- build model
- reverse everything
- generate forecasts

- Transformations help to stabilize the variance.
- For ARIMA modelling, we also need to stabilize the mean.

# Stationary?

```
gafa_stock |>
  filter(Symbol == "GOOG", year(Date) == 2018) |>
  autoplot(Close) +
  labs(y = "Google closing stock price", x = "Day")
```



# Stationary?

```
gafa_stock |>
  filter(Symbol == "GOOG", year(Date) == 2018) |>
  autoplot(difference(Close)) +
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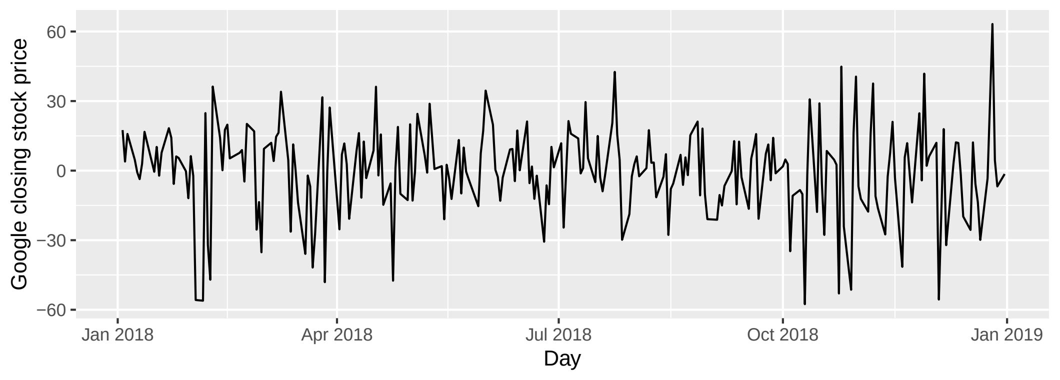
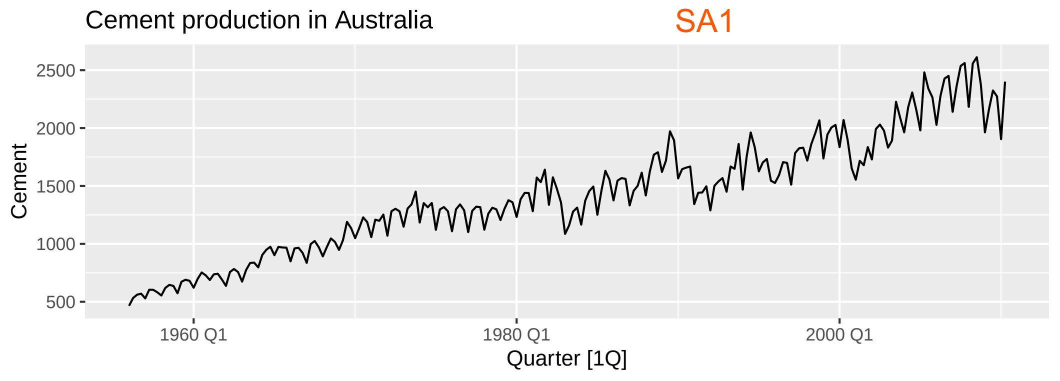


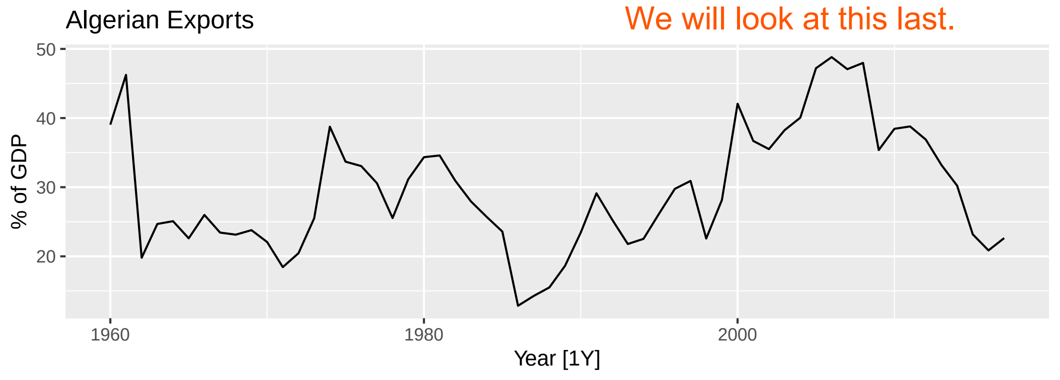
# Stationary?

```
aus_production |>
  autoplot(Cement) +
  labs(title = "Cement production in Australia")
```



# Stationary?

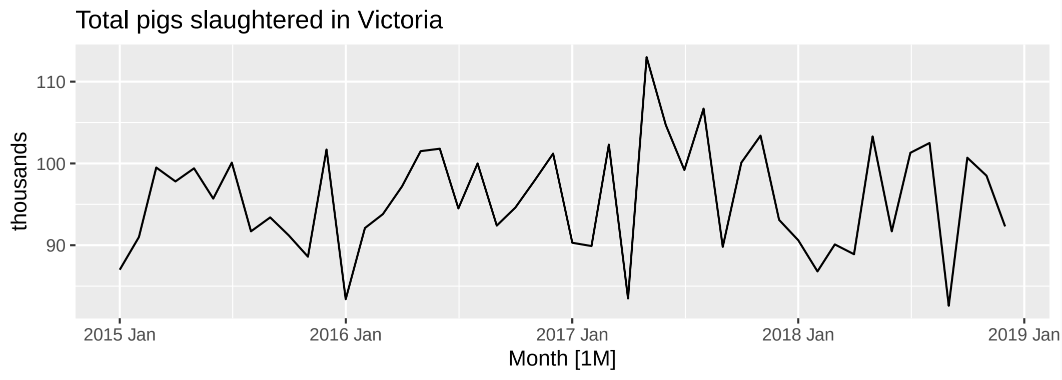
```
global_economy |>
  filter(Country == "Algeria") |>
  autoplot(Exports) +
  labs(y = "% of GDP", title = "Algerian Exports")
```




We will look at this last.

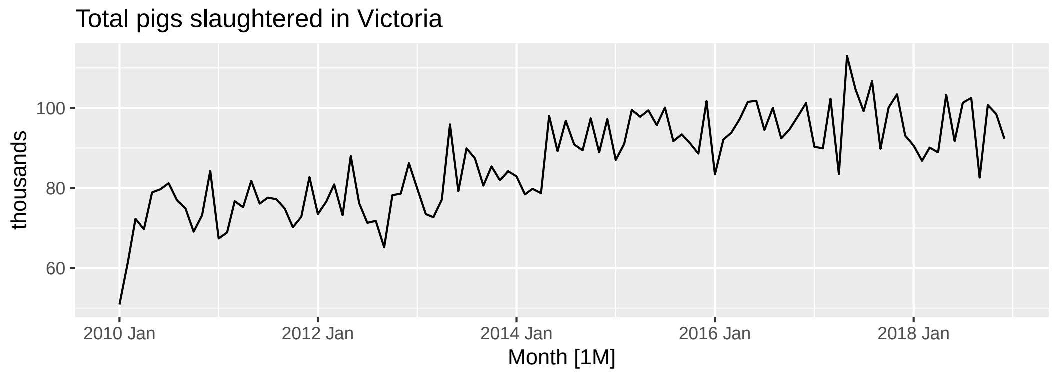
# Stationary?

```
aus_livestock |>
  filter(Animal == "Pigs", State == "Victoria", year(Month) >= 2015) |>
  autoplot(Count/1e3) +
  labs(y = "thousands", title = "Total pigs slaughtered in Victoria")
```



# Stationary?

```
aus_livestock |>
  filter(Animal == "Pigs", State == "Victoria", year(Month) >= 2010) |>
  autoplot(Count/1e3) +
  labs(y = "thousands", title = "Total pigs slaughtered in Victoria")
```



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aus_livestock |>
  filter(Animal == "Pigs", State == "Victoria") |>
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```
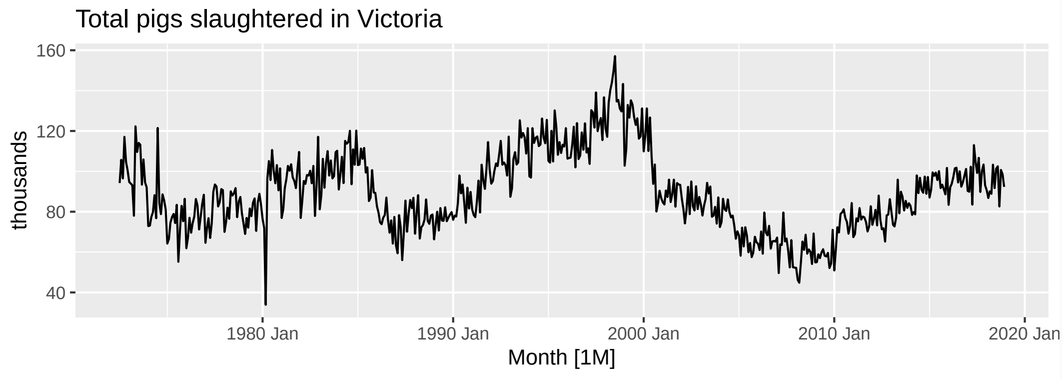
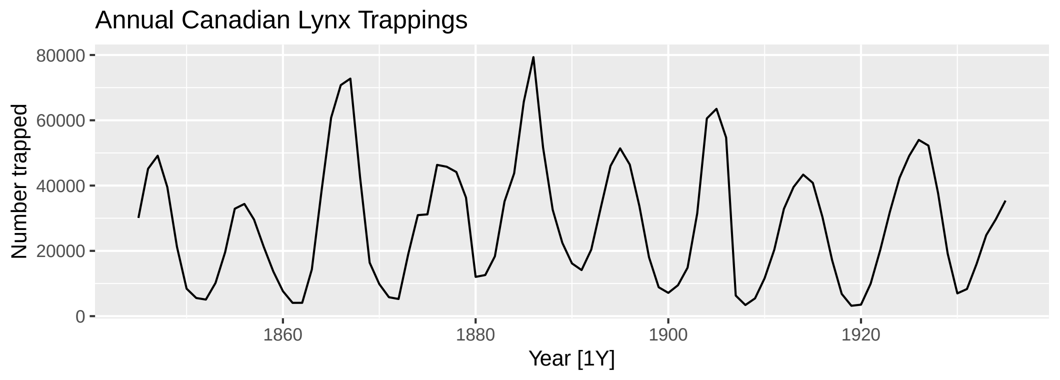



# Stationary?

```
pelt |>
  autoplot(Lynx) +
  labs(y = "Number trapped",
       title = "Annual Canadian Lynx Trappings")
```

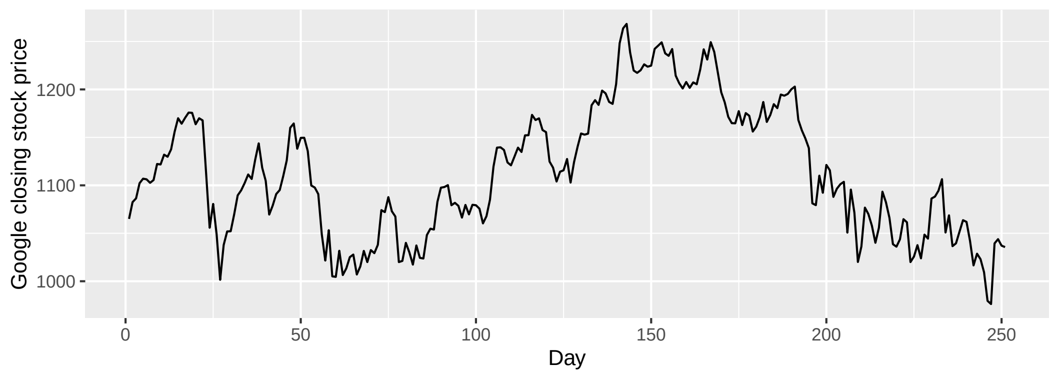


# Example: Google stock price

```
google_2018 <- gafa_stock |>
  filter(Symbol == "GOOG", year(Date) == 2018) |>
  mutate(trading_day = row_number()) |>
  update_tsibble(index = trading_day, regular = TRUE)
```

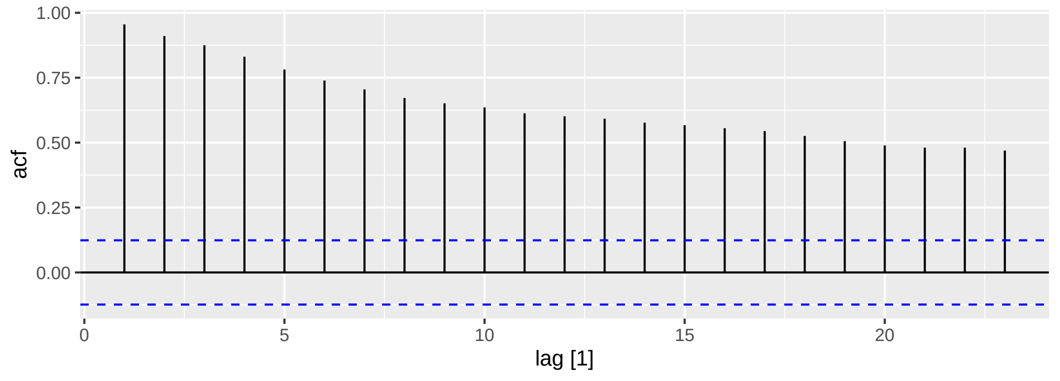
# Example: Google stock price

```
google_2018 |>
  autoplot(Close) + labs(y = "Google closing stock price", x = "Day")
```



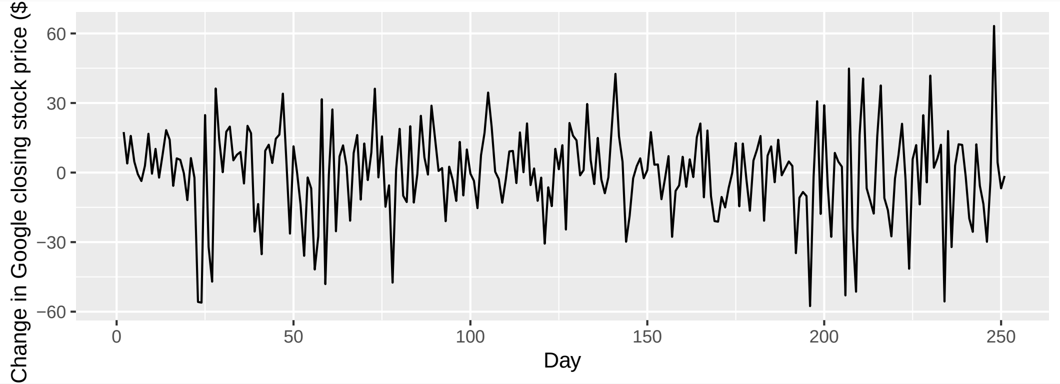
# Example: Google stock price

```
google_2018 |>
  ACF(Close) |> autoplot()
```



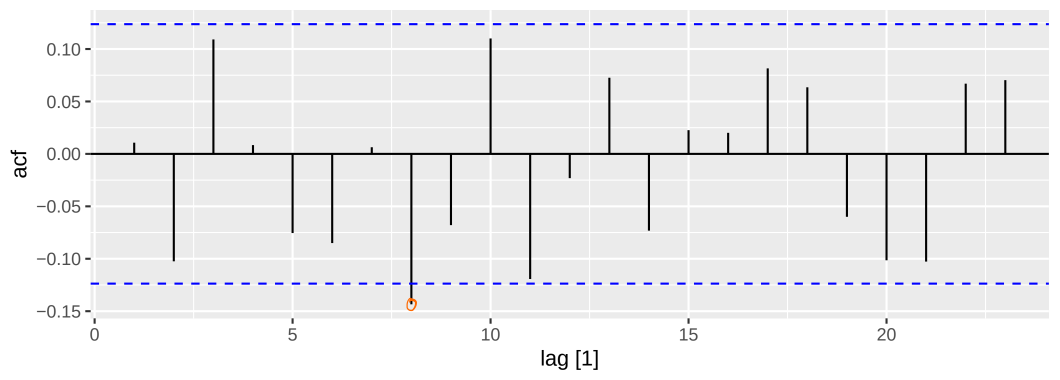
# Example: Google stock price

```r
google_2018 |>
  autoplot(difference(Close)) +
  labs(y ="Change in Google closing stock price ($USD)", x="Day")
```



# Example: Google stock price

```
google_2018 |> ACF(difference(Close)) |> autoplot()
```



# Differencing

- Differencing helps to stabilize the mean.
- The differenced series is the *change* between each observation in the original series: $y'_t = y_t - y_{t-1}$.
- The differenced series will have only $T - 1$ values since it is not possible to calculate a difference $y'_1$ for the first observation.

# Example: Google stock price

- The differences are the day-to-day changes.
- Now the series looks just like a white noise series:
  - No autocorrelations outside the 95% limits.
  - Large Ljung-Box p-value.
- Conclusion: The daily change in the Google stock price is essentially a random amount uncorrelated with previous days.

WN $\Rightarrow$ stat
stat $\not\Rightarrow$ WN

# Random walk model

- Graph of differenced data suggests the following model:

$$y_t - y_{t-1} = \varepsilon_t \qquad \text{or} \qquad y_t = y_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim NID(0, \sigma^2)$.

- Very widely used for non-stationary data.
- This is the model behind the naïve method.
- Random walks typically have:
  - long periods of apparent trends up or down.
  - Sudden/unpredictable changes in direction - stochastic trend.
- Forecast are equal to the last observation (Naive)
  - future movements are unpredictable - movements up or down are equally likely.

. The walk of a drunk

# Random walk with drift model

- If the differenced series has a non-zero mean then:

$$y_t - y_{t-1} = c + \varepsilon_t \quad \text{or} \quad y_t = c + y_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim NID(0, \sigma^2)$.

- $c$ is the non-zero average change between consecutive observations.
- If $c > 0$, $y_t$ will tend to drift upwards and vice versa.
  - Stochastic and deterministic trend.
- This is the model behind the drift method.

* Walk of a drunk pulled in some direction

* Cointegration: walk of the drunk and his dog

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# Further differencing

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- Occasionally you need to difference non-seasonal data twice.
- We seasonally difference seasonal data.

$$y'_t = y_t - y_{t-m}$$

where $m$ = number of seasons.

  - For monthly data $m = 12$, for quarterly data $m = 4$.
  - Seasonally differenced series will have $T - m$ obs.

# Seasonal random walk

If seasonally differenced data is white noise it implies:

$$y_t - y_{t-m} = \varepsilon_t \qquad \text{or} \qquad y_t = y_{t-m} + \varepsilon_t$$

- The model behind the seasonal naïve method.

# Seasonal differencing

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Common to take both seasonal and first differences. When both are applied…

- it makes no difference which is done first—the result will be the same.
- If seasonality is strong, we recommend that seasonal differencing be done first because sometimes the resulting series will be stationary and there will be no need for further first difference.

# Unit root tests

Statistical tests to determine the required order of differencing.

1. Augmented Dickey Fuller test: null hypothesis is that the data are non-stationary and non-seasonal.
2. Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test: null hypothesis is that the data are stationary and non-seasonal.
3. Other tests available for seasonal data.

# Unit root tests

Statistical tests to determine the required order of differencing.

1. Augmented Dickey Fuller test: null hypothesis is that the data are non-stationary and non-seasonal. $H_0$: non-stationary
2. Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test: null hypothesis is that the data are stationary and non-seasonal. $H_0$: stationary
3. Other tests available for seasonal data.

* In Econometrics inference is important (relies on stationarity)
* In Forecasting we don't want to difference unless we really need to
  * Control the Pr(Type I error) = $\alpha$. Reject $H_0$ while it's true.

# Seasonal differencing

STL decomposition: $y_t = T_t + S_t + R_t$

Seasonal strength $F_s = \max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(S_t+R_t)}\right)$

based on empirical evidence

If $F_s > 0.64$, do one seasonal difference.

- As $S_t \to 0$, Ratio $\to 1$, $F_s \to 0$
- As $S_t \to 1$, Ratio $\to 0$, $F_s \to 1$

# Outline

1 Stationarity and differencing

2 Backshift notation

# Backshift notation

B – a mathematical operator not a number

- First-order difference is denoted as $(1 - B)y_t$;
- Second-order difference is denoted as $(1 - B)^2 y_t$;
- Second-order difference is not the same as a second difference, which would be denoted $(1 - B^2)y_t$;
- In general, a $d$th-order difference can be written as $(1 - B)^d y_t$

# Backshift notation

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- Second-order difference is denoted as $(1 - B)^2y_t$;
- Second-order difference is not the same as a second difference, which would be denoted $(1 - B^2)y_t$; useful for 6-monthly data
- In general, a $d$th-order difference can be written as $(1 - B)^dy_t$
- A seasonal difference is denoted as $(1 - B^m)y_t$;
- A seasonal difference followed by a first difference can be written as

$$(1 - B^m)(1 - B)y_t$$